GAME PHYSICS - FORMULAS

I. Geometry

Equations of simple structures:

Line through $(0, b)$ with slope *a* : $y = ax + b$

Circle with center (a, b) and radius *r* : $(x-a)^2 + (y-b)^2 = r^2$

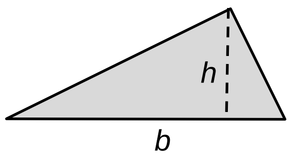
Areas and volumes:

Triangle area

$A = \frac{1}{2}bh$

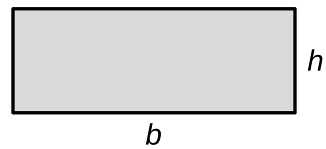
Tetrahedron volume

$V = \frac{1}{3}Ah$



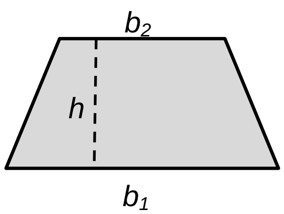
Rectangle area

$A = bh$



Trapezoid area

$A = \frac{b_1 + b_2}{2}h$

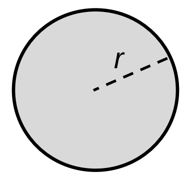


Circle area

$A = \pi r^2$

Circumference

$C = 2\pi r$

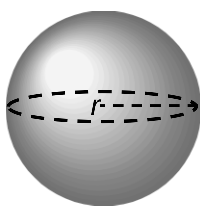


Sphere volume

$V = \frac{4}{3}\pi r^3$

Surface area

$A = 4\pi r^2$

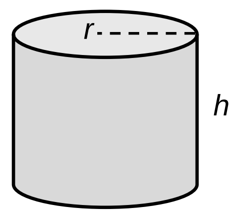


Cylinder volume

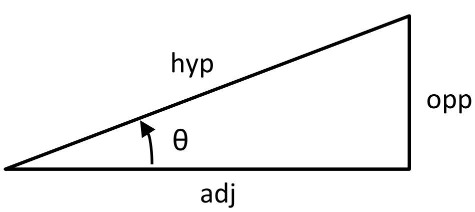
$V = \pi r^2 h$

Curved surface area

$A = 2\pi rh$



II. Trigonometry



$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

$\tan x = \frac{\sin x}{\cos x}$

For a triangle with edges *a*, *b*, *c* with respective opposite angles α, β, γ:

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

III. Differentiation

VII. A - Functions

$\boldsymbol{f(x)}$	$\boldsymbol{f'(x)}$
c	0
x	1
x^n	nx^{n-1}
c^x	$c^x \ln c$
$1/x$	$-1/x^2$
$1/x^n$	$-n/x^{n+1}$
$\sqrt{x}$	$1/(2\sqrt{x})$
$\ln x$	$1/x$
${}^c\log x$	$({}^c\log e)/x$
e^x	e^x
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$1/\cos^2 x$
$\arcsin x$	$1/\sqrt{1-x^2}$
$\arccos x$	$-1/\sqrt{1-x^2}$
$\arctan x$	$1/(1+x^2)$

VII. B - Operations

$(u+v)' = u' + v'$

$(cu)' = cu'$

$(uv)' = u'v + uv'$

$(1/u)' = -u'/u^2$

$(u/v)' = (u'v - uv')/v^2$

$(v(u))' = v'(u)\, u'$

$(e^u)' = e^u u'$

$(\ln u)' = u'/u$

$(u^\alpha)' = \alpha u^{\alpha-1} u'$

$(\sin u)' = \cos(u)\, u'$

$(\cos u)' = -\sin(u)\, u'$

IV. Notations

name	notation	name	notation
mass	m	inertia	I
time	t	time increment	Δt
position	p_o	linear displacement	Δp_o
orientation	θ	angular displacement	$\Delta\theta$
linear velocity	v	angular velocity	ω
linear acceleration	a	angular acceleration	α
force	F	torque	τ
volume	V	density	ρ
gravitational constant	$G = 6.673 \times 10^{-11}$	gravitation acceleration	$g = 9.81$

V. Moments of Inertia

Solid sphere	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$		
Hollow sphere	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$		
Solid ellipsoid	$I_{xx} = \frac{1}{5}m(b^2+c^2)$	$I_{yy} = \frac{1}{5}m(a^2+c^2)$	$I_{zz} = \frac{1}{5}m(a^2+b^2)$

Solid box	$I_{xx} = \frac{1}{12}m(h^2 + d^2)$	$I_{yy} = \frac{1}{12}m(w^2 + d^2)$	$I_{zz} = \frac{1}{12}m(w^2 + h^2)$
Solid cylinder	$I_{xx} = I_{yy} = \frac{1}{12}m(3r^2 + h^2)$	$I_{zz} = \frac{1}{2}mr^2$	
Hollow cylinder	$I_{xx} = I_{yy} = \frac{1}{12}m(6r^2 + h^2)$	$I_{zz} = mr^2$	

VI. Equations

Equations of linear motion	$v(t + \Delta t) = v(t) + a\Delta t$ $\bar{v} = \frac{v(t + \Delta t) + v(t)}{2}$ $\Delta p_o = {}^1/_2\,(v(t + \Delta t) + v(t))\Delta t$ $\Delta p_o = v(t)\Delta t + {}^1/_2\,a\,\Delta t^2$ $v(t + \Delta t)^2 = v(t)^2 + 2a\Delta p_o$
Newton's Second Law of Linear Motion	$F_{net} = m * a$
Newton's Law of Gravitation	$F_g = F_{A \to B} = -F_{B \to A} = G\frac{m_A m_B}{r^2}u_{AB}$
Weight force	$W = m * g$
Static and kinetic friction forces	$F_s = \mu_s * F_N$ and $F_k = \mu_k * F_N$
Drag forces	$F_{D_{high}} = -\frac{1}{2} * \rho * v^2 * C_d * A$ and $F_{D_{low}} = -b * v$
Buoyancy force	$F_B = \rho * g * V$
Spring force	$F_s = -K(l - l_0)$
Damper force	$F_C = -C(v_A - v_B)$
Work	$W = F * \Delta p_o$
Translational kinetic energy	$E_K = \frac{1}{2}\,m\,v^2$
Work-Energy theorem	$W = \Delta E_K = E_K(t + \Delta t) - E_K(t)$

Potential energy	$E_P = m * g * h$
Linear momentum	$p = m * v$
Impulse force	$F\Delta t = \Delta p$
Equations of angular motion	$\omega(t+\Delta t) = \omega(t) + \alpha\Delta t$ $\bar{\omega} = \frac{\omega(t+\Delta t) + \omega(t)}{2}$ $\Delta\theta = {}^1\!/_2\,(\omega(t+\Delta t) + \omega(t))\Delta t$ $\Delta\theta = \omega(t)\Delta t + {}^1\!/_2\,\alpha\,\Delta t^2$ $\omega(t+\Delta t)^2 = \omega(t)^2 + 2\alpha\Delta\theta$
Tangential acceleration	$a_t = \alpha * r$
Centripetal acceleration	$a_n = r\omega^2$
Torque	$\tau = r \times F$
Newton's Second Law of Angular Motion	$\tau_{net} = I * \alpha$
Rotational kinetic energy	$E_{Kr} = \frac{1}{2} * I * \omega^2$
Conservation of mechanical energy	$E_{Kt}(t+\Delta t) + E_P(t+\Delta t) + E_{Kr}(t+\Delta t)$ $= E_{Kt}(t) + E_P(t) + E_{Kr}(t) + E_O$
Angular momentum	$L = I * \omega$
Impulse torque	$\tau\Delta t = \Delta L$
Mass	$m = \int_V \rho\, dV$
Center of Mass	$COM = \frac{1}{m}\int_V \rho(p) * p\, dV$
Moments of inertia	$I_{xx} = \int (y^2 + z^2) dm$ $I_{yy} = \int (z^2 + x^2) dm$ $I_{zz} = \int (x^2 + y^2) dm$

Products of inertia	$I_{xy} = I_{yx} = \int (xy)dm$ $I_{xz} = I_{zx} = \int (xz)dm$ $I_{yz} = I_{zy} = \int (yz)dm$
Parallel axis theorem	$I_v = I_{COM} + mr^2$
PD controller	$\tau = k_p(\theta_d - \theta) + k_v(\dot{\theta}_d - \dot{\theta})$
Midpoint integration method	$p_o(t + \Delta t) = p_o(t) + \Delta t * v\left(t + \frac{\Delta t}{2}, p_o + \frac{\Delta t}{2} v(t, p_o)\right)$
Improved Euler's integration method	$v_1 = v(t) + \Delta t * a(t, v)$ $v_2 = v(t) + \Delta t * a(t + \Delta t, v_1)$ $v(t + \Delta t) = \frac{v_1 + v_2}{2}$
Runge-Kutta order 4 integration method	$v_1 = \Delta t * a(t, v(t))$ $v_2 = \Delta t * a\left(t + \frac{\Delta t}{2}, v(t) + \frac{1}{2} v_1\right)$ $v_3 = \Delta t * a\left(t + \frac{\Delta t}{2}, v(t) + \frac{1}{2} v_2\right)$ $v_4 = \Delta t * a(t + \Delta t, v(t) + v_3)$ $v(t + \Delta t) = v(t) + \frac{v_1 + 2v_2 + 2v_3 + v_4}{6}$
Verlet integration method	$p_o(t + \Delta t) = 2p_o(t) - p_o(t - \Delta t) + \Delta t^2 p_o''(t)$
Backward Euler's integration method	$p_o(t + \Delta t) = p_o + \Delta t * v(t + \Delta t)$
Semi-implicit integration method	$v(t + \Delta t) = v(t) + \Delta t * a(t)$ $p_o(t + \Delta t) = p_o + \Delta t * v(t + \Delta t)$
Coefficient of restitution	$C_R = -\frac{(v_{A+} - v_{B+}) \cdot n}{(v_{A-} - v_{B-}) \cdot n}$
Collision impulse (without rotation)	$j = -(1 + C_r)(v_{A-} - v_{B-}) \cdot n / \left(\frac{1}{m_A} + \frac{1}{m_B}\right)$
Collision impulse (with rotation)	$j = -(1 + C_r)(v_{A-} - v_{B-}) \cdot n \; /$ $\left(\frac{1}{m_A} + \frac{1}{m_B}\right) + \left[\left(I_A^{-1}(r_A \times n)\right) \times r_A + \left(I_B^{-1}(r_B \times n)\right) \times r_B\right] \cdot n$
Linear velocity resolution	$v_+ = v_- + \frac{j}{m} n$

Angular velocity resolution	$\omega_{+} = \omega_{-} + I^{-1}(r \times (j * n))$
Stress	$\sigma = F/A$
Strain	$\epsilon = \Delta L/L$
Young’s modulus	$Y = \frac{linear\ \sigma}{linear\ \epsilon}$
Shear modulus	$S = \frac{planar\ \sigma}{planar\ \epsilon}$
Bulk modulus	$B = \frac{volume\ \sigma}{volume\ \epsilon}$
Poisson’s ratio	$\nu = -\frac{d\ transverse\ \sigma}{d\ axial\ \sigma}$